

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Thursday 21 June 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1.
$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}.$$

(a) Find the values of the constants A , B and C . (4)

(b) (i) Hence find $\int f(x) \, dx$.

(ii) Find $\int_1^2 f(x) \, dx$, leaving your answer in the form $a + \ln b$, where a and b are constants. (6)

2.

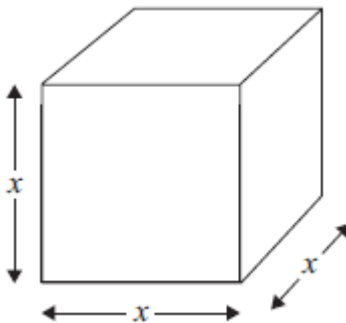


Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.

At time t seconds, the length of each edge of the cube is x cm, and the volume of the cube is $V \text{ cm}^3$.

(a) Show that $\frac{dV}{dx} = 3x^2$. (1)

Given that the volume, $V \text{ cm}^3$, increases at a constant rate of $0.048 \text{ cm}^3 \text{ s}^{-1}$,

(b) find $\frac{dx}{dt}$ when $x = 8$, (2)

(c) find the rate of increase of the total surface area of the cube, in $\text{cm}^2 \text{ s}^{-1}$, when $x = 8$. (3)

3.
$$f(x) = \frac{6}{\sqrt{9-4x}}, \quad |x| < \frac{9}{4}.$$

(a) Find the binomial expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . Give each coefficient in its simplest form. (6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x , up to and including the term in x^3 , of

(b) $g(x) = \frac{6}{\sqrt{9+4x}}, \quad |x| < \frac{9}{4},$ (1)

(c) $h(x) = \frac{6}{\sqrt{9-8x}}, \quad |x| < \frac{9}{8}.$ (2)

4. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y \cos^2 x}.$$

(5)

5. The curve C has equation

$$16y^3 + 9x^2y - 54x = 0.$$

(a) Find $\frac{dy}{dx}$ in terms of x and y . (5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$. (7)

6.

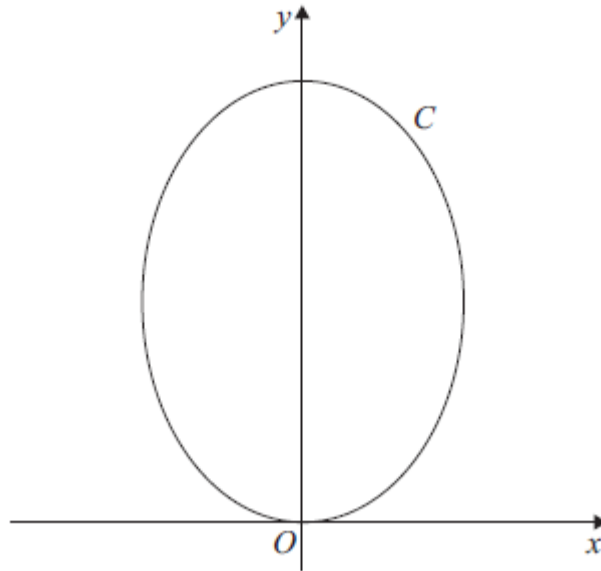


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3} \sin 2t, \quad y = 4 \cos^2 t, \quad 0 \leq t \leq \pi.$$

(a) Show that $\frac{dy}{dx} = k\sqrt{3} \tan 2t$, where k is a constant to be determined.

(5)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

(c) Find a cartesian equation of C .

(3)

7.

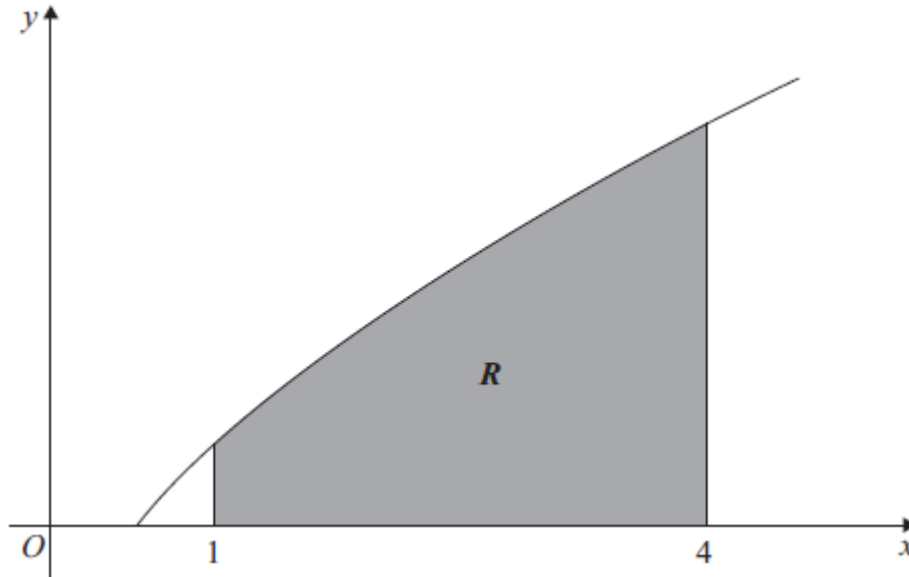


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and the lines $x = 1$ and $x = 4$.

(a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R , giving your answer to 2 decimal places. (4)

(b) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$. (4)

(c) Hence find the exact area of R , giving your answer in the form $a \ln 2 + b$, where a and b are exact constants. (3)

8. Relative to a fixed origin O , the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B .

(a) Find the vector \overrightarrow{AB} . (2)

(b) Find a vector equation for the line l . (2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l . Given that the vector \overrightarrow{CP} is perpendicular to l ,

(c) find the position vector of the point P . (6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
<p>1. (a)</p> <p>(b)</p> <p>(i)</p> <p>(ii)</p>	$1 = A(3x-1)^2 + Bx(3x-1) + Cx$ $x \rightarrow 0 \quad (1 = A)$ $x \rightarrow \frac{1}{3} \quad 1 = \frac{1}{3}C \Rightarrow C = 3$ <p>Coefficients of x^2</p> $0 = 9A + 3B \Rightarrow B = -3$ $\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$ $= \ln x - \frac{3}{3} \ln(3x-1) + \frac{3}{(-1)3} (3x-1)^{-1} \quad (+C)$ $\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$ $\int_1^2 f(x) dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$ $= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$ $= \ln \frac{2 \times 2}{5} + \dots$ $= \frac{3}{10} + \ln \left(\frac{4}{5} \right)$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(4)</p> <p>M1 A1ft A1ft</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(6)</p> <p>[10]</p>
<p>2. (a)</p> <p>(b)</p> <p>(c)</p>	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2 \quad *$ $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{0.048}{3x^2}$ <p>At $x = 8$, $\frac{dx}{dt} = \frac{0.048}{3(8^2)} = 0.00025 \quad (\text{cms}^{-1})$</p> $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$ $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} = 12x \left(\frac{0.048}{3x^2} \right)$ <p>At $x = 8$ $\frac{dS}{dt} = 0.024 \quad (\text{cm}^2 \text{s}^{-1})$</p>	<p>cs0</p> <p>B1</p> <p>(1)</p> <p>M1</p> <p>A1</p> <p>(2)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>[6]</p>

Question Number	Scheme	Marks
<p>3. (a)</p>	$f(x) = \dots (\dots - \dots x)^{-\frac{1}{2}}$ $= 6 \times 9^{-\frac{1}{2}} (\dots)$ <p style="text-align: right;">$\frac{6}{9^{\frac{1}{2}}}, \frac{6}{3}, 2$ or equivalent</p> $= \dots \left(1 + \left(-\frac{1}{2}\right)(kx) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(kx)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(kx)^3 + \dots \right)$ $= 2 \left(1 + \frac{2}{9}x + \dots \right)$ <p style="text-align: right;">or $2 + \frac{4}{9}x$</p> $= 2 + \frac{4}{9}x + \frac{4}{27}x^2 + \frac{40}{729}x^3 + \dots$	<p>M1</p> <p>B1</p> <p>M1; A1ft</p> <p>A1</p> <p>A1 (6)</p>
<p>(b)</p>	$g(x) = 2 - \frac{4}{9}x + \frac{4}{27}x^2 - \frac{40}{729}x^3 + \dots$	<p>B1ft (1)</p>
<p>(c)</p>	$h(x) = 2 + \frac{4}{9}(2x) + \frac{4}{27}(2x)^2 + \frac{40}{729}(2x)^3 + \dots$ $\left(= 2 + \frac{8}{9}x + \frac{16}{27}x^2 + \frac{320}{729}x^3 + \dots \right)$	<p>M1 A1 (2)</p>
<p>4.</p>	$\int y \, dy = \int \frac{3}{\cos^2 x} \, dx$ <p style="text-align: right;">Can be implied. Ignore integral signs</p> $= \int 3 \sec^2 x \, dx$ $\frac{1}{2}y^2 = 3 \tan x \quad (+C)$ $y = 2, x = \frac{\pi}{4}$ $\frac{1}{2}2^2 = 3 \tan \frac{\pi}{4} + C$ <p>Leading to</p> $C = -1$ $\frac{1}{2}y^2 = 3 \tan x - 1$ <p style="text-align: right;">or equivalent</p>	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (5)</p> <p>[5]</p>

Question Number	Scheme	Marks
5. (a)	Differentiating implicitly to obtain $\pm ay^2 \frac{dy}{dx}$ and/or $\pm bx^2 \frac{dy}{dx}$ $48y^2 \frac{dy}{dx} + \dots - 54 \dots$ $9x^2 y \rightarrow 9x^2 \frac{dy}{dx} + 18xy$ or equivalent $(48y^2 + 9x^2) \frac{dy}{dx} + 18xy - 54 = 0$ $\frac{dy}{dx} = \frac{54 - 18xy}{48y^2 + 9x^2} \left(= \frac{18 - 6xy}{16y^2 + 3x^2} \right)$	M1 A1 B1 M1 A1 (5)
(b)	$18 - 6xy = 0$ Using $x = \frac{3}{y}$ or $y = \frac{3}{x}$ $16y^3 + 9\left(\frac{3}{y}\right)^2 y - 54\left(\frac{3}{y}\right) = 0$ or $16\left(\frac{3}{x}\right)^3 + 9x^2\left(\frac{3}{x}\right) - 54x = 0$ Leading to $16y^4 + 81 - 162 = 0$ or $16 + x^4 - 2x^4 = 0$ $y^4 = \frac{81}{16}$ or $x^4 = 16$ $y = \frac{3}{2}, -\frac{3}{2}$ or $x = 2, -2$ Substituting either of their values into $xy = 3$ to obtain a value of the other variable. $\left(2, \frac{3}{2}\right), \left(-2, -\frac{3}{2}\right)$ both	M1 M1 M1 A1 A1 M1 A1 (7) [12]

Question Number	Scheme	Marks
6. (a)	$\frac{dx}{dt} = 2\sqrt{3} \cos 2t$ $\frac{dy}{dt} = -8 \cos t \sin t$ $\frac{dy}{dx} = \frac{-8 \cos t \sin t}{2\sqrt{3} \cos 2t}$ $= -\frac{4 \sin 2t}{2\sqrt{3} \cos 2t}$ $\frac{dy}{dx} = -\frac{2}{3}\sqrt{3} \tan 2t \quad \left(k = -\frac{2}{3}\right)$	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (5)</p>
(b)	<p>When $t = \frac{\pi}{3}$ $x = \frac{3}{2}, y = 1$ can be implied</p> $m = -\frac{2}{3}\sqrt{3} \tan\left(\frac{2\pi}{3}\right) (= 2)$ $y - 1 = 2\left(x - \frac{3}{2}\right)$ $y = 2x - 2$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1 (4)</p>
(c)	$x = \sqrt{3} \sin 2t = \sqrt{3} \times 2 \sin t \cos t$ $x^2 = 12 \sin^2 t \cos^2 t = 12(1 - \cos^2 t) \cos^2 t$ $x^2 = 12\left(1 - \frac{y}{4}\right) \frac{y}{4}$ <p style="text-align: right;">or equivalent</p>	<p>M1</p> <p>M1 A1 (3)</p> <p>[12]</p>

Question Number	Scheme	Marks															
7. (a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">x</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td style="text-align: center;">3</td> <td style="text-align: center;">4</td> </tr> <tr> <td style="text-align: center;">y</td> <td style="text-align: center;">$\ln 2$</td> <td style="text-align: center;">$\sqrt{2} \ln 4$</td> <td style="text-align: center;">$\sqrt{3} \ln 6$</td> <td style="text-align: center;">$2 \ln 8$</td> </tr> <tr> <td></td> <td style="text-align: center;">0.6931</td> <td style="text-align: center;">1.9605</td> <td style="text-align: center;">3.1034</td> <td style="text-align: center;">4.1589</td> </tr> </table>	x	1	2	3	4	y	$\ln 2$	$\sqrt{2} \ln 4$	$\sqrt{3} \ln 6$	$2 \ln 8$		0.6931	1.9605	3.1034	4.1589	M1
	x	1	2	3	4												
	y	$\ln 2$	$\sqrt{2} \ln 4$	$\sqrt{3} \ln 6$	$2 \ln 8$												
	0.6931	1.9605	3.1034	4.1589													
$\text{Area} = \frac{1}{2} \times 1(\dots)$ $\approx \dots (0.6931 + 2(1.9605 + 3.1034) + 4.1589)$ $\approx \frac{1}{2} \times 14.97989 \dots \approx 7.49$	B1 M1 A1 (4)																
(b)	$\int x^{\frac{1}{2}} \ln 2x \, dx = \frac{2}{3} x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3} x^{\frac{3}{2}} \times \frac{1}{x} \, dx$ $= \frac{2}{3} x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3} x^{\frac{1}{2}} \, dx$ $= \frac{2}{3} x^{\frac{3}{2}} \ln 2x - \frac{4}{9} x^{\frac{3}{2}} \quad (+C)$	M1 A1 M1 A1 (4)															
(c)	$\left[\frac{2}{3} x^{\frac{3}{2}} \ln 2x - \frac{4}{9} x^{\frac{3}{2}} \right]_1^4 = \left(\frac{2}{3} 4^{\frac{3}{2}} \ln 8 - \frac{4}{9} 4^{\frac{3}{2}} \right) - \left(\frac{2}{3} \ln 2 - \frac{4}{9} \right)$ $= (16 \ln 2 - \dots) - \dots \quad \text{Using or implying } \ln 2^n = n \ln 2$ $= \frac{46}{3} \ln 2 - \frac{28}{9}$	M1 M1 A1 (3) [11]															

Question Number	Scheme	Marks
8. (a)	$\vec{AB} = \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	M1 A1 (2)
(b)	$\mathbf{r} = \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	M1 A1ft (2)
(c)	$\vec{CP} = \begin{pmatrix} 10-2t \\ 2+t \\ 3+t \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 7-2t \\ t-10 \\ t \end{pmatrix}$ $\begin{pmatrix} 7-2t \\ t-10 \\ t \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -14 + 4t + t - 10 + t = 0$	M1 A1
	Leading to $t = 4$	A1
	Position vector of P is $\begin{pmatrix} 10-8 \\ 2+4 \\ 3+4 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$	M1 A1 (6)
		[10]